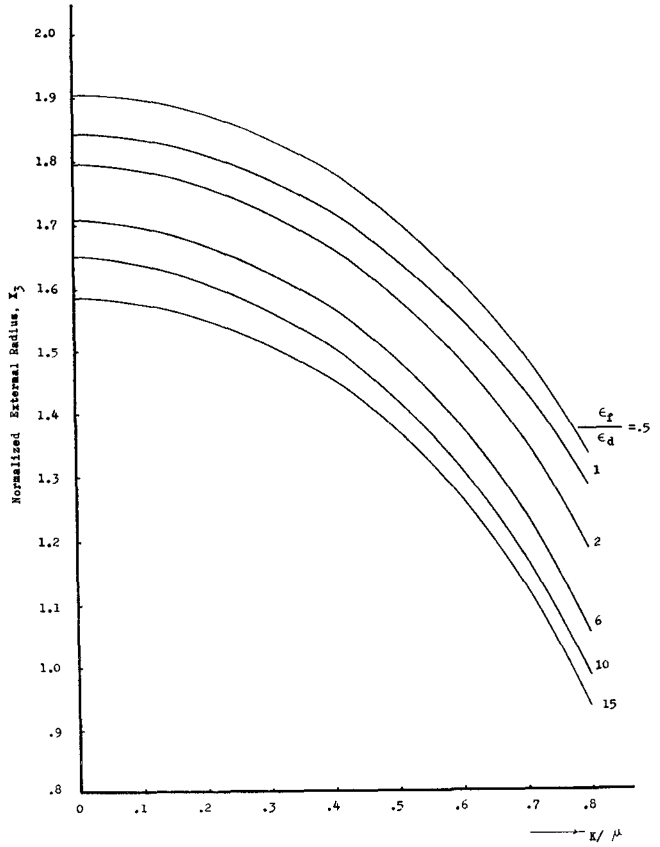

Fig. 4. Normalized external radius x_3 for circulation as a function of K/μ for $R = 1.1$ and different values of ϵ_f/ϵ_d.

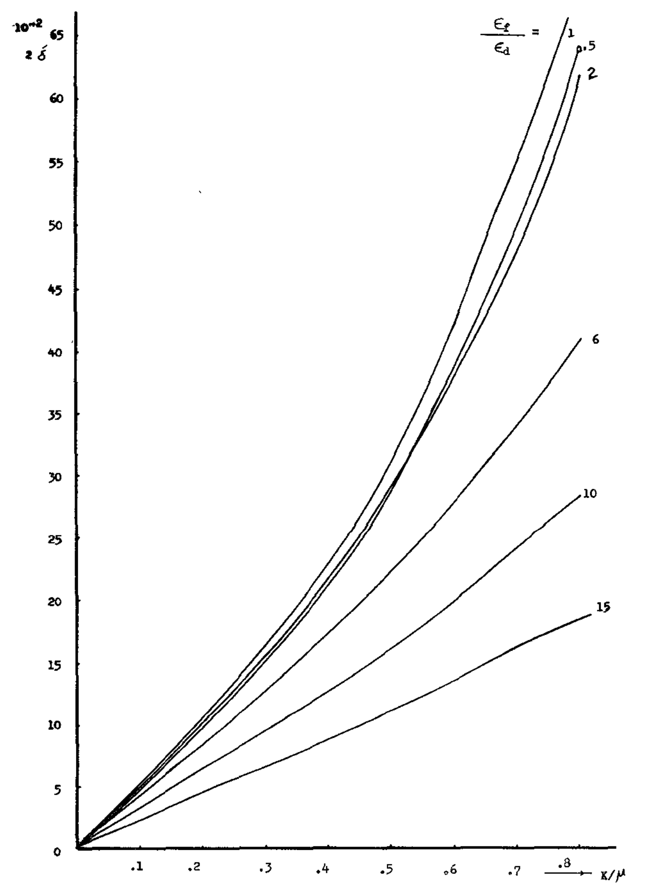

Fig. 5. The splitting ratio $2\delta' = (x_3^- - x_3^+)/x_{3_{0ir}}$ as a function of K/μ for $R = 1.1$ and different values of ϵ_f/ϵ_d.

IV. Conclusions

The effect of the width and the dielectric material of the nonmagnetic gap on the operation of latching stripline circulators has been studied. Numerical results show that in the typical range of design the external normalized radius and the bandwidth are slightly affected by the gapwidth. Generally speaking the bandwidth decreases by changing the dielectric constant of the ceramic ring from that of the ferrite. The normalized external radius decreases by decreasing the dielectric constant of the ceramic ring. The bandwidth is not sensitive to the variations in ϵ_f/ϵ_d up to the value 2.

References

[1] W. W. Siekanowicz and W. A. Schilling, "A new type of latching switchable ferrite junction circulator," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-16, pp. 177–183, Mar. 1968.
[2] H. Bosma, "On stripline Y-circulation at UHF," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-12, pp. 61–72, Jan. 1964.
[3] C. E. Fay and R. L. Comstock, "Operation of the ferrite junction circulator," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-13, pp. 15–27, Jan. 1965.
[4] W. W. Siekanowicz, R. W. Paglione, and T. E. Walsh, "A latching ring-and-post ferrite waveguide circulator," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-18, pp. 212–216, Apr. 1970.
[5] M. E. El-Shandwily, A. A. Kamal, and E. A. F. Abdallah, "General field theory treatment of *H*-plane waveguide junction circulators," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-21, pp. 392–403, June 1973.

Curved-Rim Open Resonators

A. CONSORTINI

Abstract—**Field configurations and resonant frequencies are analytically derived for some low-loss modes of a Fabry–Perot (FP) open resonator having curved rims along the edges. Since the low-loss modes are limited by a caustic surface, the problem can be simply treated by neglecting the diffraction due to the finite dimensions of the mirrors. The results are compared with those obtained by numerically solving the integral equation of the open cavity.**

I. Introduction

As is well known, the losses of a class of open resonators are so low that field configurations and resonant frequencies can be obtained, with a good approximation, without taking into account the effects of the diffraction due to the finite dimensions of the mirrors. In general, this class includes those open cavities whose low-loss modes are limited by a caustic surface [1]. Typical examples are the curved stable resonators [1], [2], the so-called flat-roof resonator [3], the quasi-corner resonator [4], and some types of rimmed resonators [5], [6].

Another type of cavity, where the low-order modes are expected to be represented by fields bounded by a caustic surface, is that represented in Fig. 1 constituted by a Fabry–Perot (FP) resonator having curved rims along the edges. In the present short paper, mode configuration and resonances of it are determined by neglecting the diffractional effects.

As usual, the problem is reduced to the investigation of the infinite-strip case. The curved-rim sections are assumed to join the flat portion of the mirrors continuously (Fig. 1).

II. Theory

With reference to rectangular coordinates x,y,z with the origin at the center of the cavity, Fig. 1, the problem is to find a solution of the wave equation, for instance, the electric field E parallel to y, which satisfies the boundary conditions on the mirrors. We will

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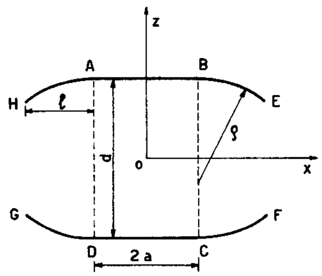

Fig. 1.

proceed by choosing two different solutions satisfying the boundary conditions in the central region $ABCD$, and in a lateral region, $BEFC$, respectively, and then by matching them along an appropriate line [3], [4], [6].

In the central portion of the cavity, a solution of the wave equation, satisfying the boundary conditions on the walls, can be written

$$E_c = \cos k_z z \begin{cases} \cos (k_x x) \\ \sin (k_x x) \end{cases} \tag{1}$$

where

$$k_z d = q\pi \tag{2}$$

and q is a very large odd integer. For low-order modes, which we are interested in, k_z is of the order of magnitude of the free-space wavenumber k, and therefore $k_x \ll k_z$. Moreover, $k_x a$ is of the order of a multiple of $\pi/2$. It is to be expected that the presence of the rim alters the value of k_x, but not its order of magnitude.

The factors $\cos (k_x x)$ and $\sin (k_x x)$ of (1) correspond to even and odd modes, respectively.

The solution E_l of the wave equation in the region $BEFC$, satisfying the boundary conditions at BE and FC (and having a caustic surface in this region), can be looked for in the form of two Gaussian beams with the axis coincident with BC, propagating in the directions z and $-z$, respectively, and having BE and CF as wavefronts. In order to obtain low-loss modes it is necessary to utilize the Gaussian beams of lowest order, so that E_l can be written as

$$E_l = A \frac{w_0}{w} \exp(-r^2/w^2) \cos\left[kz - \Phi + \frac{kr^2}{2R}\right] \tag{3}$$

where

$$r = x - a \tag{4}$$

$$k = (k_x^2 + k_z^2)^{1/2} \simeq k_z + \frac{1}{2}\frac{k_x^2}{k_z}. \tag{5}$$

Moreover [7],

$$w = w_0\left[1 + \left(\frac{\lambda z}{\pi w_0^2}\right)^2\right]^{1/2}, \qquad \lambda = 2\pi/k \tag{6}$$

$$R = z\left[1 + \left(\frac{\pi w_0^2}{\lambda z}\right)^2\right] \tag{7}$$

and

$$\Phi = \arctan\left(\frac{\lambda z}{\pi w_0^2}\right). \tag{8}$$

Finally, w_0 is a constant (the beam waist) which is determined by means of the condition that the mirror surfaces coincide with two wavefronts. By requiring that for $|z| = d/2$ the radius of curvature $|R|$ of the wavefront be equal to the radius ρ of the rim, one obtains from (7)

$$w_0^2 = \frac{d}{k}\left(\frac{2\rho}{d} - 1\right)^{1/2}. \tag{9}$$

Recall that the existence of Gaussian beams requires $w_0 \gg \lambda$. It appears from this equation that the caustic surface exists only if $\rho > d/2$, as is well known from the theory of curved-mirror open resonators.

It is also well known that in the region $r < w$ the field of a Gaussian beam presents an oscillating behavior; the field becomes vanishingly small when $r > w$. The line $r(z) = w(z)$ represents the caustic.

The two fields E_c and E_l must be matched along the line separating the two regions of validity of the two solutions (1) and (3). Note that since the plane and curved mirrors are tangent to one another, the two regions are not sharply limited by the line $x = a$. Therefore, there is no physical reason for the matching to occur along BC, as would happen if the mirrors were not tangent to one another. The line of the matching, of equation $r = r_0(z)$, is an unknown of our problem.

The matching requires the following conditions to be fulfilled along the line $r_0(z)$:

$$[E_c]_{r=r_0(z)} = [E_l]_{r=r_0(z)} \tag{10}$$

$$\left[\frac{\partial E_c}{\partial x}\right]_{r=r_0(z)} = \left[\frac{\partial E_l}{\partial x}\right]_{r=r_0(z)} \tag{11}$$

$$\left[\frac{\partial E_c}{\partial z}\right]_{r=r_0(z)} = \left[\frac{\partial E_l}{\partial z}\right]_{r=r_0(z)} \tag{12}$$

In general, these conditions cannot be satisfied everywhere along the line of the matching with the simple expressions (1) and (3) for E_c and E_l, respectively. As an approximation we will require that they are satisfied near the x axis. This is equivalent to approximating the line $r = r_0(z)$ with the straight line $r = r_0(z = 0) = r_0$. In the same approximation, conditions (10) and (12) obviously coincide. In this case the quantities Φ, w, and R appearing in (3) can be replaced by their approximate expressions valid for small values of z. For small values of z the matching conditions, on account of (1) and (3), give for even modes

$$A = \frac{\cos[k_x(a + r_0)]}{\exp[-r_0^2/w_0^2]} \tag{13}$$

$$k_x \tan[k_x(a + r_0)] = \frac{2r_0}{w_0^2} \tag{14}$$

$$k = k_z\left[1 + \frac{2}{k_z^2 w_0^2}\left(1 - \frac{r_0^2}{w_0^2}\right)\right], \qquad r_0 < w_0 \tag{15}$$

and analogous expressions for odd modes.

Equation (15) represents the resonance condition. It has been obtained from (12) by also recalling that $w_0 > \lambda$ and, consequently, by neglecting terms of higher order with respect to the last term in square parenthesis.

A comparison of (15) and (5) yields

$$k_x^2 = \frac{4}{w_0^2}\left(1 - \frac{r_0^2}{w_0^2}\right). \tag{16}$$

After some manipulation, by also using (16), (14) can be replaced by

$$\cos^2[k_x(a + r_0)] = 1 - \frac{r_0^2}{w_0^2}. \tag{17}$$

Since $k_x a$ is of the order of a multiple of $\pi/2$, it appears from (17) that $r_0 \simeq w_0$. The solution of this equation, on account also of (14), can be looked for in the form:

$$k_x(a + r_0) = (m + 1)\frac{\pi}{2}(1 - \epsilon), \qquad m = 0,2\cdots \tag{18}$$

with $0 < \epsilon \ll 1$. For odd modes one derives the same equation, but with m odd. Equation (18) introduced into (16) allows us to derive an approximate value r_0' of r_0, by neglecting ϵ with respect to unity and r_0 with respect to the mirror aperture a:

$$r_0' = w_0\left[1 - \frac{(m + 1)^2\pi^2}{16}\left(\frac{w_0}{a}\right)^2\right]^{1/2}. \tag{19}$$

This result implies that the second term in brackets be much smaller than unity, so that our results will be valid for low values of m. If one introduces (19) into (15) it appears that in this approximation the resonance frequencies of the cavity are the same as those of the FP resonator constituting the central part of the resonator, as expected.

Let us now pass to a better approximation. Introduction of (18) into (17) yields

$$\sin^2\left[(m+1)\frac{\pi}{2}\epsilon\right] = \left(1 - \frac{r_0^2}{w_0^2}\right). \tag{20}$$

One can write, from (20), in a first approximation,

$$\epsilon \simeq \frac{2}{\pi(m+1)}\left(1 - \frac{r_0^2}{w_0^2}\right)^{1/2}. \tag{21}$$

Introduction of (18) into (16), on account of (21), gives

$$\left[(m+1)\frac{\pi}{2} - \left(1 - \frac{r_0^2}{w_0^2}\right)^{1/2}\right]^2 = \frac{4(a+r_0)^2}{w_0^2}\left(1 - \frac{r_0^2}{w_0^2}\right). \tag{22}$$

In order to solve this equation let us replace r_0 in the factor $(a + r_0)$ by the value r_0' obtained with the zero-order approximation. It is then an easy matter to find

$$1 - \frac{r_0^2}{w_0^2} = \frac{(m+1)^2\pi^2 w_0^2}{4[2(a+r_0') + w_0]^2}. \tag{23}$$

The resonance condition becomes

$$kd = q\pi + \frac{(m+1)^2\pi}{4N_a}\frac{a^2}{(2a + w_0 + 2r_0')^2} \tag{24}$$

where $N_a = a^2/\lambda d$ denotes the Fresnel number of the central part of the resonator and

$$k_x = \frac{(m+1)\pi}{2a + w_0 + 2r_0'}. \tag{25}$$

Equation (24) or (25) together with (9) and (19) gives the resonance condition; (1) and (3) furnish the mode patterns. When the mirror aperture $2a$ becomes large with respect to w_0 and r_0', these equations reduce to those of the FP resonator. An inspection of the above equations indicates that our results can be expressed in terms of nondimensional quantities by dividing each length by λ.

III. Conclusion

The results derived above were obtained with a procedure including many limitations and approximations.

Recall that the procedure is valid for resonators having a rim curvature $\rho > d/2$, so that a caustic surface exists. Moreover, the approximations made in deriving our formulas, which require

$$w_0 > \lambda \tag{26}$$

and

$$(m+1)\frac{\pi}{2}\epsilon \ll 1 \tag{27}$$

put further limits to ρ:

$$1 + \left(\frac{2\pi\lambda}{d}\right)^2 < \frac{2\rho}{d} \ll \left[\frac{32N_a}{(m+1)^2\pi}\right]^2 + 1. \tag{28}$$

Finally, we recall that the matching conditions (10)–(12) cannot be satisfied everywhere along the line of the matching with the simple expressions (1) and (3). In particular, the field near the mirrors seems to have a rather complicated pattern owing to the finiteness of the mirrors [9], [10].

In spite of the number of simplifications, the treatment turns out to work satisfactorily in a large range of values of the parameters. An idea of the accuracy of the results can be given by a comparison with the results obtained by numerically solving the integral equation of the open cavity. Fig. 2 shows the phase shift defined as $\Delta q = kd - q\pi$ for four modes, in the particular case $a/\lambda = 20$, $d/\lambda = 100$, plotted versus λ/ρ. Solid lines refer to the present theory. For the open resonator (dashed lines), the rim width has a value $l/\lambda = 5$. The curves for the values $m = 0$ and $m = 1$ are reported from [8, fig. 11] (Note, however, that in that figure the scale of phase shifts is affected by a misprint so that a correction factor 10^{-1} must be introduced). The curves of the modes $m = 2$ and $m = 3$ were evaluated by using a procedure described elsewhere [11].

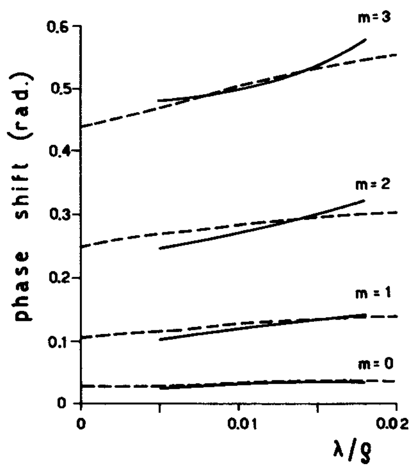

Fig. 2. $d/\lambda = 100$; $a = 20\lambda$; $N_a = 4$; --- $l = 5\lambda$.

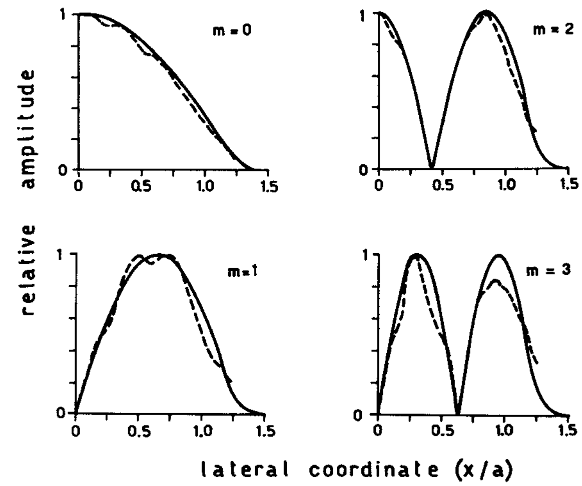

Fig. 3. $d/\lambda = 100$; $a = 20\lambda$; $N_a = 4$; --- $l = 5\lambda$; $\lambda/\rho = 0.01$.

Mode patterns for $m = 0,1,2,3$ are reported in Fig. 3, in the case $\lambda/\rho = 0.01$ (solid line). The radius of the curved mirrors corresponds to a confocal geometry, which does not, however, give rise to the minimum of the losses versus λ/ρ, as can be seen from [7, fig. 11]. The central portion of the resonator corresponds to $x/a \leq 1$; the rim extends from $x/a = 1$ to $x/a = 1.25$. The beam waist is $w_0 = 3.99\lambda$ and the value of r_0', where the matching is to be made, is $r_0' = 3.94\lambda$ for the first even mode and $r_0' = 3.79\lambda$ for the first odd mode. The dashed lines of Fig. 3 were obtained with the numerical evaluation by means of the procedure described in [11]. The agreement between theoretical and numerical results is quite good, in spite of the fact that w_0 is not larger than λ and that the open curved mirrors have a dimension only a little larger than w_0. Even better agreement would be obtained for larger values of λ/ρ.

These simple examples confirm the utility of the procedures of neglecting the losses for a first insight in the behavior of open resonators where the field is confined by a caustic surface.

Moreover, the knowledge of approximate expressions of the fields may be the starting point for a more complete analysis also including the losses of the resonator.

References

[1] L. A. Vainstein, "Open resonators and open waveguides," *Sov. Radio Eng. (Moscow)*, 1966.
[2] G. D. Boyd and H. Kogelnik, "Generalized confocal resonator theory," *Bell Syst. Tech. J.*, vol. 41, p. 1347, 1962.
[3] G. Toraldo di Francia, "Flat-roof resonator," *Appl. Opt.*, vol. 4, p. 1267, 1965.

[4] L. Ronchi, "Low-loss modes and resonances in a quasi-90°-roof mirror resonator," *Appl. Opt.*, vol. 12, p. 93, 1973.
[5] A. B. Manenkov, "Study of an open resonator with concentrated field," in *High Power Electronics* (in Russian), vol. 5, *Izv. Akad. Nauk SSSR (Moscow)*, p. 64, 1968.
[6] A. Consortini, "Sloped-rim open resonators," *Appl. Opt.*, vol. 12, p. 1011, 1973.
[7] H. Kogelnik and T. Li, "Laser beams and resonators," *Proc. IEEE*, vol. 54, pp. 1312–1329, Oct. 1966.
[8] P. F. Checcacci, A. Consortini, and A. M. Scheggi, "Effect of mirror rims on modes and losses of a planar Fabry-Perot resonator," *Appl. Opt.*, vol. 10, p. 1363, 1971.
[9] F. Pasqualetti and L. Ronchi, "Quasi-stationary wave patterns inside infinite-strip open resonators," *Appl. Opt.*, vol. 10, p. 2488, 1971.
[10] M. Born and E. Wolf, *Principles of Optics*. London: Pergamon, 1959, p. 574.
[11] A. Consortini, and F. Pasqualetti, "An analysis of the modes of the Fabry-Perot resonator," *Opt. Acta*, to be published.

Circularly Polarized Equalizer Networks

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Abstract—**Characteristics of the cross slot coupling aperture applicable to circularly polarized equalizer networks is presented. This coupling mechanism is analyzed and experimental results indicate good agreement between theory and practice. Extension of the single-cavity unit to multiple direct-coupled dual-mode equalizer networks is also discussed.**

Introduction

In recent years as microwave communications system requirements have become more sophisticated, the design of multiplexing bandpass networks together with the necessary equalization circuitry has attracted an increasing amount of attention. The choice of applicable equalizer networks has usually relied upon either single-mode cavities [1]–[4], using circulators or hybrids, or dual orthogonal mode [5] circularly polarized cavity networks. In general, the single dominant mode equalizer networks exhibit more dissipation loss, and the achievable isolation is limited due to the inherent circuit characteristics of the circulator or hybrid. Furthermore, with circulators or hybrids the weight will increase and most likely will be more expensive, especially if temperature compensation of the ferromagnetic material of circulators is required. In all cases the theory of reflection-type commensurate transmission-line all-pass networks has been utilized to analytically describe the behavior of these networks [6], [7].

This short paper describes an improved approach for realizing circularly polarized equalizer networks whereby use of the cross is employed instead of the more conventional circle or square-shaped iris as the coupling mechanism between the main transmission line and the appropriate cavity circuitry. The primary advantage of cross aperture coupling is that the return loss or match is much better than that exhibited by either the circle or square iris given identical coupling. The VSWR for the circular or square aperture can be significantly improved through the use of screw tuning in the main line. However, this matching will restrict the allowable number of cascaded equalizer units that can be tandem connected due to the frequency sensitivity of the spacing between each tuning screw, i.e., the reactive interaction effects. The inherent superior scattering characteristic of the cross will additionally prove valuable when utilized in multiple cascaded directional channel circular waveguide filter networks.

In this short paper, application of existing theory for circularly polarized single-cavity resonators is made with emphasis on the use of cross slot coupling for realizing microwave equalizer networks. Various properties of the cross are quantitatively defined. In addition, direct-coupled multiple-cavity equalizers are also discussed.

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Cross Aperture Coupling

A. General

If a pair of crossed slots couples energy into a cylindrical waveguide cavity, configured such that the dominant TE_{11}^{0} mode can be orthogonally doubly degenerate, then the cavity supporting a resonant circularly polarized wave can be coupled to the rectangular waveguide $TE_{10}^{\square}$ mode in such a manner that an all-pass single-resonator equalizer circuit is realized. Fig. 1 denotes this circularly polarized equalizer network with the appropriate dimensional parameters.

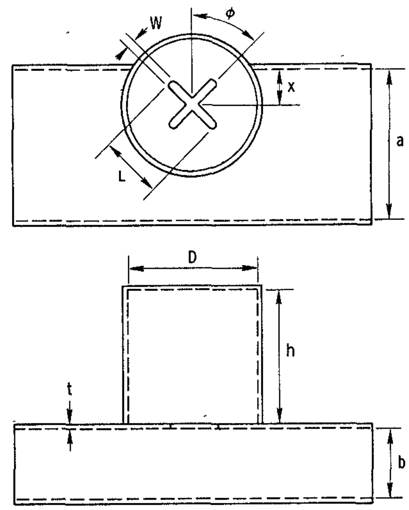

Fig. 1. CP equalizer using cross slot coupling.

The transverse location x is unique to the operation of circularly polarized equalizers in that only one location exists which provides circular polarization (CP). In fact, a proper x position allows the simultaneous achievement of minimum scattering with good CP. The amount of coupling is controlled by both the length and the width of the slots. The design of such circularly polarized cavity structures has been considered for directional filters [8] with circular hole coupling irises. Both the peak magnitude of the time delay and the time delay response as a function of frequency are determined by a single parameter, the external Q of the resonant cavity, which for singly loaded equalizer cavities is twice that of doubly terminated single-cavity filters [9].

B. Cross Location

The time delay response of the equalizer is completely specified by the power coupling factor and the resonant frequency of the cavity. As denoted in [10, p. 239], the power coupling coefficient is related to the square of the magnitude of the transfer scattering coefficient. Circular polarization will exist when the coupling to each orthogonal mode is equal from which the cross location is defined as

$$x = \frac{a}{\pi} \tan^{-1} \left[\frac{\lambda g}{2a} \tan^2 \phi \right]. \tag{1}$$

In general, the cross angle ϕ is set to about 45° with respect to the transverse axis of the rectangular waveguide. However, this angle is not critical and can be easily compensated for by a slight adjustment of the x position of the slots. In addition, it should be noted that as the angle ϕ is allowed to increase slightly above 45°, the optimum cross position will move a small distance toward the center of the rectangular waveguide which, in turn, will permit larger coupling values to exist by virtue of longer slots possible before the proximity of the side wall interferes.

It is necessary to determine a proper x position to achieve a match into the CP equalizer network. Fig. 2 shows the sensitivity of the match to the x position for an equalizer with a center frequency of